5. Strack, J.D.L. *Groundwater Mechanics.* Prentice Hall, Englewood Cliffs, 1989.

**I. I. Sakhaev (Kazan), C. Lomp (Porto), M. F. Nasrutdinov (Kasan)**

## ON PROJECTIVE MODULES OF FINITE DUAL GOLDI DIMENSION

Let $R$ be an associative ring with unit, $P$ will be a left unital $R$-module and $J(P)$ denote the Jacobson's radical of $P$.

A submodule $N$ of $P$ is said *small* in $P$ if for every submodule $U$ of $P$ the equation $N + U = P$ involves $U = P$. A module $P$ is said to be hollow if $P \neq 0$ and every proper submodule of $P$ is small in $P$. A module $P$ is said to have *finite hollow dimension* (or *finite dual Goldie dimension*) if there exists an exact sequence

$$P \xrightarrow{g} \bigoplus_{i=1}^{n} H_i \longrightarrow 0$$

where all $H_i$ are hollow and the kernel of $R$–homomorphism $g$ is small in $P$. Then $n$ is called the hollow dimension of $P$ and we write $hdim(P) = n$.

In the paper [1] it was formulated the question: *Is every projective $R$-module $P$ with semilocal endomorphism ring $Hom_R(P, P)$ finitely generated?*

If $P$ is projective $R$-module and $Hom_R(P, P)$ is semilocal ring then by the theorem 3.10 [1] we have $hdim_R(P) < \infty$. We have proved:

**Theorem** *Let $P$ be a projective $R$-module and $Hom_R(P, P)$ is semilocal ring. Then the following conditions are equivalent:*
*(a) $P$ is finitely generated.*
*(b) $hdim_R(P) = hdim_R(P/J(P))$.*

Research of I.I. Sakhaev and M.F. Nasrutdinov is supported by RFFI grant 99-01-00469.

REFERENCES

1. Lomp C. *On semilocal modules and rings*// Communication Algebra. – 1999. - V. 27. – P. 1921–1935.

**Th. Lutz (Stuttgart, Germany)**

## SUBSONIC AND TRANSONIC AIRFOIL DESIGN APPLYING NUMERICAL OPTIMIZATION TECHNIQUES

The specific design of airfoils is one of the classical tasks of aerodynamics. Since the airfoil characteristics are directly dependent on the inviscid pressure distribution the application of inverse calculation methods is obvious. The numerical airfoil optimization offers an alternative to the inverse design and attracts increasing interest. With this approach an automated search for an optimal solution with respect to a user-specified objective function is performed. An overview about recent results on subsonic and transonic airfoil optimizations will be given.

The objective of the subsonic airfoil optimizations was to design natural laminar flow airfoils which show minimized drag for a specified range of the Reynolds number and the lift coefficient.

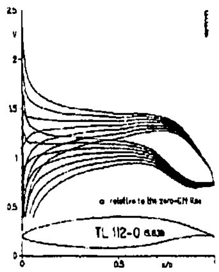

Fig.1

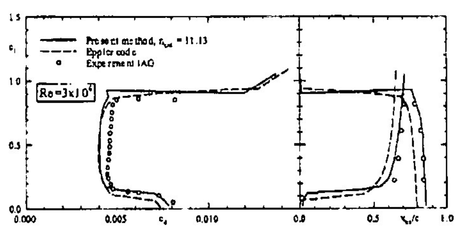

Fig.2

To this purpose an efficient aerodynamic model was coupled with a hybrid optimizer [2]. Contrary to the usual approach the airfoil is not parameterized by geometric shape functions. Instead, the inverse conformal mapping procedure according to Eppler [1] was applied to generate the airfoil shape and to evaluate the inviscid